Invariant Pattern Recognition using Dual-tree Complex Wavelets and Fourier Features

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Abstract

In this paper, we propose an invariant descriptor for pattern recognition by using the dual-tree complex wavelet and the Fourier transform. The approximate shift-invariant property of the dual-tree complex wavelets and the good property of the Fourier transform make our descriptor a very attractive choice for invariant pattern recognition. Experiments show that our proposed descriptor is very robust to Gaussian white noise and it achieves high recognition rates.

Keywords: wavelets, dual-tree complex wavelets, Fourier transform, feature extraction, pattern recognition.

1 Introduction

Feature extraction is a crucial step in invariant pattern recognition ([1] - [18]). Many invariant descriptors use the Fourier transform and the wavelet transform to extract invariant features. The Fourier transform has been a powerful tool for pattern recognition ([2] - [7]). One important property of the Fourier transform is that a shift in the time domain causes no change in the magnitude spectrum. This can be used to extract invariant features in pattern recognition. Translation invariance of a 2-D pattern can be achieved by taking the magnitude spectrum of the 2-D Fourier transform of the pattern. Rotation invariance can be obtained by performing 1-D Fourier transform in the angular direction in polar coordinates. Scale invariance can be accomplished by taking the logarithm of the polarized image and then conducting a 1-D Fourier transform along the radial direction.

Wavelet transforms have also been proved to be very popular and effective in pattern recognition. Here we briefly review some of the papers for pattern recognition using wavelets. Bui et al. [8] proposed an invariant descriptor by using the orthonormal shell and the Fourier transform. The descriptor is invariant to translation, rotation, and scaling. Chen and Bui [9] invented an invariant descriptor by using a combination of Fourier transform and wavelet transform. They polarize the pattern first, and then perform 1-D wavelet transform along the radial direction and 1-D Fourier transform in the angular direction. Lee et al. [10] proposed a scheme for multiresolution recognition of unconstrained handwritten numerals using the wavelet transform and a simple multilayer cluster neural network. The wavelet features of handwritten numeral at two decomposition levels are fed into the multilayer cluster neural network. Wunsch and Laine [11] gave a descriptor by extracting wavelet features from the outer contour of the handwritten characters and feeding the features into neural networks. Their experiments were done for handprinted characters. Chen et al. [12] developed a descriptor by using multiwavelets and neural networks. The multiwavelet features are also extracted from the outer contour of the handwritten numerals and fed into neural networks. This descriptor gives higher recognition rate than the one given in [11] for handwritten numeral recognition. Khalil and Bayoumi [13] investigated to use wavelet modulus maxima for invariant 2-D pattern recognition. Tieng and Boles [14] used wavelet zero-crossing to recognize 2-D patterns. Tao et al. proposed a technique for feature extraction by using wavelet and fractal [15]. Shen and Ip [16] presented a set of wavelet moment invariants for the classification of seemingly similar objects with subtle differences. Tieng and Boles considered wavelet-based affine invariant pattern recognition in [17] and [18].

In this paper, we propose a novel feature extraction technique for pattern recognition.The pattern is first normalized so that it is translation invariant and scale invariant.A transformation from the Cartesian coordinate system to the polar coordinate system is then performed. We conduct a 1-D Fourier transform along the angular direction and take the magnitude, and a 1-D dual-tree complex wavelet transform along the radial direction. The descriptor thus obtained is invariant to translation, rotation, and scaling. Experimental results show that the descriptor achieves higher recognition rates than the descriptor developed in [9] for recognizing noisy patterns.

The organization of this paper is as follows. Section 2 reviews the dual-tree complex wavelet and its shift invariant property. Section 3 proposes a novel feature extraction technique for pattern recognition by using the dual-tree complex wavelet and the Fourier transform. Section 4 conducts some experiments for pattern recognition by considering different rotation angles and noise levels. Finally Section 5 gives the conclusions and future work to be done.

2 The Dual-Tree Complex Wavelet

It is well known that the ordinary discrete wavelet transform is not shift invariant because of the decimation operation during the transform. A small shift in the input signal can cause very different output wavelet coefficients. This is the main limitation of wavelet in pattern recognition. One way of overcoming this is to do the wavelet transform without decimation. The drawback of this approach is that it is computationally inefficient, especially in multiple dimensions.

Kingsbury ([19] - [21]) introduced a new kind of wavelet transform, called the dual-tree complex wavelet transform, that exhibits approximate shift invariant property and improved angular resolution. The success of the transform is because of the use of filters in two trees, a and b. He proposed a simple delay of one sample between the level 1 filters in each tree, and then the use of alternate odd-length and even-length linear-phase filters. As he pointed out that there are some difficulties in the odd/even filter approach. Therefore, he proposed a new $Q-shift$ dual-tree [22] where all the filters beyond level 1 are even length. The filters in the two trees are just the time-reverse of each other, as are the analysis and reconstruction filters. The new filters are shorter than before, and the new transform still satisfies the shift invariant property and good directional selectivity in multiple dimensions. As will be shown later, this dual-tree complex wavelet can be successfully used in invariant feature extraction for pattern recognition.

3 Pattern Recognition by using a Combination of Fourier and Dual-tree Complex Wavelet Features

A good pattern recognition descriptor should extract features that are invariant to translation, rotation, and scaling. In this paper, we use the available techniques to preprocess the pattern so that it is invariant to translation and scaling. The rotation invariance can be achieved by our newly developed descriptor below. We polarize the pattern the same way as the descriptor developed in [9]. In order to achieve rotational invariance, we apply the 1-D Fourier transform along the axis of the polar angle to obtain the magnitude of its spectrum. Since the spectrum magnitude of the Fourier transform of circularly shifted signals are the same, we obtain features that are rotation invariant. Because the dual-tree complex wavelet coefficients have approximate shift-invariant property and represent pattern features at different resolution levels, we apply the dual-tree complex wavelet transform along the radial axis. We use the features obtained in this way to recognize the pattern at different resolution levels.

The steps of the our newly proposed descriptor can be given as follows:

1. Find the centroid of the pattern $f(x, y)$ and move the centroid to the center of the image.

2. Normalize the pattern so that it is scale invariant.

3. Transform $f(x, y)$ into polar coordinate system to obtain $g(r, \theta)$.

4. Conduct 1-D Fourier transform on $g(r, \theta)$ along the axis of polar angle θ and obtain its spectrum magnitude:
$$G(r, \phi) = |FT_\theta(g(r, \theta))|.$$

5. Apply 1-D dual-tree complex wavelet transform on $G(r, \phi)$ along the radial axis r:
$$WF(r, \phi) = DTWT_r(G(r, \phi)).$$

6. Use the daul-tree complex wavelet coefficients to query the pattern feature database at different resolution levels.

Fig. 1 shows the steps of the new descriptor. It is easy to see that this descriptor is similar to that developed in [9]. The only difference is that we replaced the 1-D wavelet transform in [9] with the 1-D dual-tree complex wavelet transform. The approximate shift-invariant property of the dual-tree complex wavelet transform makes this newly proposed descriptor a very good choice for invariant pattern recognition. Experimental results show that this descriptor is an excellent choice for pattern recognition, and it is also robust to Gaussian white noise.

4 Experimental Results

We tested our proposed descriptor by using a database of 85 printed Chinese characters in our experiments. The database is shown in Fig. 2. Every character is represented by 64×64 pixels. Our major concern in our experiments is the performance of the descriptor on rotation. For each character, we tested nine rotation angles 30^o, 60^o, 90^o, 120^o, 150^o, 180^o, 210^o, 240^o and 270^o. Daubechies-4 wavelet is used in this experiment. We test the performance of our proposed descriptor on noisy data. The noisy images with different orientations are generated by adding Gaussian white noise to the noise-free images. The signal to noise ratio (SNR) is defined as

$$\mathrm{SNR} = \frac{\sqrt{\sum_{i,j}(f_{i,j} - avg(f))^2}}{\sqrt{\sum_{i,j}(n_{i,j} - avg(n))^2}}$$

where f is the noise-free image, n is the added white noise, and $avg(f)$ is the average value of the image f. We conduct experiments for SNR=20, 15, 10, 5, 4, 3, 2, 1, and 0.5. Fig. 3 shows the noisy patterns with theses SNR's. The experimental results are listed in Table 1. It is clear that our proposed descriptor is very robust to Gaussian white noise.

We compare the recognition rates of our proposed descriptor with those of [9] under the noisy environment. Chen and Bui [9] developed an invariant descriptor by using a combination of the Fourier transform and the wavelet transform. They polarize the pattern first, and then perform a 1-D wavelet transform along the radial direction and a 1-D Fourier transform along the angular direction. We give the recognition rates of the descriptor developed in [9] in Table 2. By looking at the recognition rates in Table 1 and Table 2, we can see that our proposed descriptor produces much better recognition results than the descriptor developed in [9] under the noisy environment. In fact, at SNR=0.5, it is very difficult to recognize the patterns even by human eyes. However, our proposed descriptor can do an excellent job in this case. Note that we do not perform any denoising procedure on the noisy pattern. Our proposed descriptor extracts invariant features from the noisy pattern directly and it achieves very high recognition rates. This indicates that our proposed descriptor is very robust to Gaussian white noise even when the noise level is high.

5 Conclusions

In this paper, we present an invariant descriptor for pattern recognition by combining Fourier spectrum with the dual-tree complex wavelets. The scale variance and translation variance are eliminated by standard normalization techniques. Our descriptor is also rotational invariant. The approximate shift-invariant property of the dual-tree complex wavelet transform makes our new descriptor outperform the descriptor developed in [9] for recognizing noisy patterns. Experimental results show that our proposed descriptor is a reliable choice for pattern recognition. It is evident that this descriptor is very robust to Gaussian white noise. Future work will be done by introducing SVM or neural networks to the new descriptor for the classification phase.

Acknowledgments

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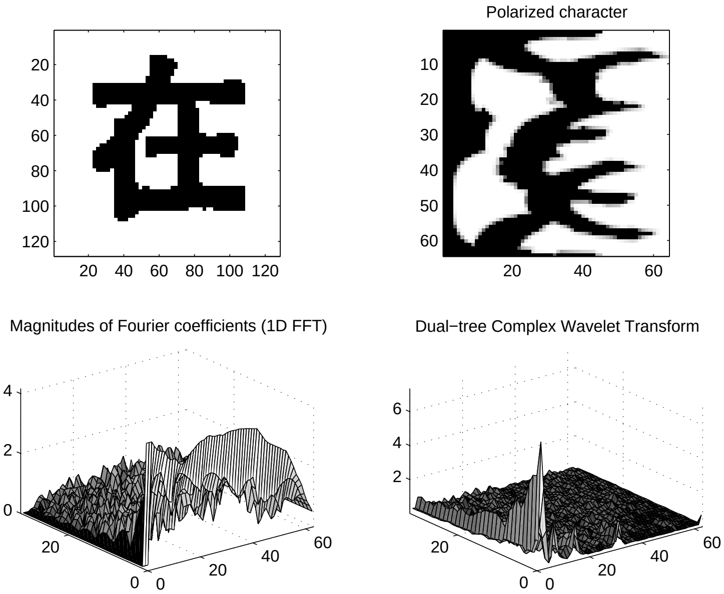

Figure 1: The original character, its polarization, its Fourier magnitude spectrum, and the dual-tree complex wavelet coefficients.

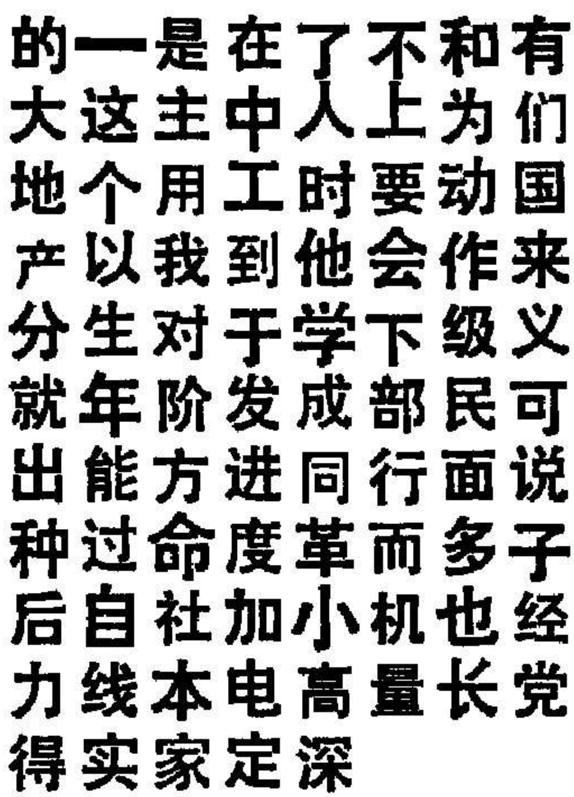

Figure 2: The Chinese character database used in this paper.

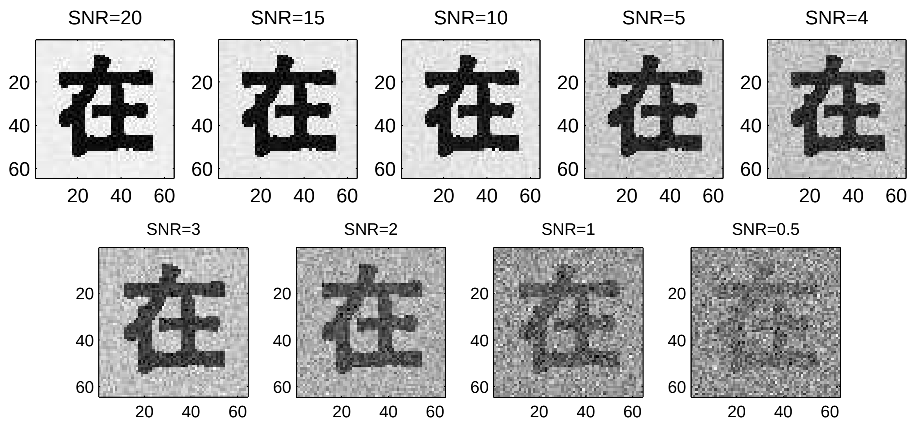

Figure 3: The noisy patterns with SNR = 20, 15, 10, 5, 4, 3, 2, 1, and 0.5, respectively.

Different	*Rotation*								
SNR	30^o	60^o	90^o	120^o	150^o	180^o	210^o	240^o	270^o
20	100	100	100	100	100	100	100	100	100
15	100	100	100	100	100	100	100	100	100
10	100	100	100	100	100	100	100	100	100
5	100	100	100	100	100	100	100	100	100
4	100	100	100	100	100	100	100	100	100
3	100	100	100	100	100	100	100	100	100
2	100	100	100	100	100	100	100	100	100
1	96.47	96.47	100	94.12	92.94	96.47	94.12	95.29	98.82
0.5	74.12	69.41	65.88	55.29	61.18	61.18	61.18	61.18	67.06

Table 1: The recognition rates of our proposed descriptor for different SNR's.

Different	*Rotation*								
SNR	30^o	60^o	90^o	120^o	150^o	180^o	210^o	240^o	270^o
20	100	100	100	100	100	100	100	100	100
15	100	100	100	100	100	100	100	100	100
10	100	100	100	100	100	100	100	100	100
5	100	100	100	100	100	100	100	100	100
4	100	100	100	100	100	100	100	100	100
3	100	100	100	100	100	100	100	100	100
2	100	100	100	97.65	97.65	98.82	100	98.82	100
1	90.59	89.41	89.41	85.88	84.71	87.06	87.06	84.71	89.41
0.5	60.00	55.29	56.47	44.71	50.59	48.24	45.88	47.06	60.00

Table 2: The recognition rates of [9] for different SNR's.