Complex Ridgelets for Image Denoising

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Abstract

In this paper, we propose a novel image denoising method by incorporating the dual-tree complex wavelets into the ordinary ridgelet transform. The approximate shift invariant property of the dual-tree complex wavelet and the high directional sensitivity of the ridgelet transform make the new method a very good choice for image denoising. We apply the digital complex ridgelet transform to the denoising of some standard images embedded in white noise. A simple hard thresholding of the complex ridgelet coefficients is used. Experimental results show that the new method is better than *VisuShrink*, the ordinary ridgelet image denoising, and $wiener2$ filter that is available in the Matlab Image Processing Toolbox. Complex ridgelets could be applied to curvelet image denoising as well.

Keywords: Image denoising, wavelets, ridgelets, complex ridgelets.

1 Introduction

Wavelet transforms have been successfully used in many scientific fields such as image compression, image denoising, signal processing, computer graphics, and pattern recognition, to name only a few. Donoho and his coworkers pioneered a wavelet denoising scheme by using soft thresholding and hard thresholding. This approach appears to be a good choice for a number of applications. This is because a wavelet transform can compact the energy of the image to only a small number of large coefficients and the majority of the wavelet coefficients are very small so that they can be set to zero. The thresholding of the wavelet coefficients can be done at only the detail wavelet decomposition subbands. We keep a few low frequency wavelet subbands untouched so that they are not thresholded. It is well known that Donoho's method offers the advantages of smoothness and adaptation. However, as Coifman and Donoho pointed out, this algorithm exhibits visual artifacts: Gibbs phenomena in the neighbourhood of discontinuities. Therefore, they propose in [1] a translation invariant (TI) denoising scheme to suppress such artifacts by averaging over the denoised signals of all circular shifts. The experimental results in [1] confirm that single TI wavelet denoising performs better than the non-TI case. Bui and Chen [2] extended this TI scheme to the multiwavelet case and they found that TI multiwavelet denoising gave better results than TI single wavelet denoising. Cai and Silverman [3] proposed a thresholding scheme by taking the neighbour coefficients into account. Their experimental results showed apparent advantages over the traditional term-by-term wavelet denoising. Chen and Bui [4] extended this neighbouring wavelet thresholding idea to the multiwavelet case. They claimed that neighbour multiwavelet denoising outperforms neighbour single wavelet denoising for some standard test signals and real-life images. Chen et al. [5] proposed an image denoising scheme by considering a square neighbourhood in the wavelet domain. Chen et al. [6] also tried to customize the wavelet filter and the threshold for image denoising. Experimental results show that these two methods produce better denoising results.

The ridgelet transform was developed over several years to break the limitations of the wavelet transform. The 2D wavelet transform of images produces large wavelet coefficients at every scale of the decomposition. With so many large coefficients, the denoising of noisy images faces a lot of difficulties. We know that the ridgelet transform has been successfully used to analyze digital images ([7] - [14]). Unlike wavelet transforms, the ridgelet transform processes data by first computing integrals over different orientations and locations. A ridgelet is constant along the lines $x_1 cos\theta + x_2 sin\theta = constant$. In the direction orthogonal to these ridges it is a wavelet.

Ridgelets have been successfully applied in image denoising recently. In this paper, we combine the dual-tree complex wavelet in the ridgelet transform and apply it to image denoising. The approximate shift invariance property of the dual-tree complex wavelet and the good property of the ridgelet make our method a very good method for image denoising. Experimental results show that by using dual-tree complex ridgelets, our algorithms obtain higher Peak Signal to Noise Ratio (PSNR) for all the denoised images with different noise levels.

The organization of this paper is as follows. In Section 2, we explain how to incorporate the dual-tree complex wavelets into the ridgelet transform for image denoising. Experimental results are conducted in Section 3. Finally we give the conclusion and future work to be done in section 4.

2 Image Denoising by using Complex Ridgelets

Discrete ridgelet transform provides near-ideal sparsity of representation of both smooth objects and of objects with edges([7] - [14]). It is a near-optimal method of denoising for Gaussian noise. The ridgelet transform can compress the energy of the image into a smaller number of ridgelet coefficients. On the other hand, the wavelet transform produces many large wavelet coefficients on the edges on every scale of the 2D wavelet decomposition. This means that many wavelet coefficients are needed in order to reconstruct the edges in the image.

We know that approximate Radon transforms for digital data can be based on discrete fast Fourier transform. The ordinary ridgelet transform can be achieved as follows [10]:

1. Compute the 2D FFT of the image.
2. Substitute the sampled values of the Fourier transform obtained on the square lattice with sampled values on a polar lattice.
3. Compute the 1D inverse FFT on each angular line.
4. Perform the 1D scalar wavelet transform on the resulting angular lines in order to obtain the ridgelet coefficients.

It is well known that the ordinary discrete wavelet transform is not shift invariant because of the decimation operation during the transform. A small shift in the input signal can cause very different output wavelet coefficients. In order to overcome this problem, Kingsbury ([15] - [18]) introduced a new kind of wavelet transform, called the dual-tree complex wavelet transform, that exhibits approximate shift invariant property and improved angular resolution. Since the scalar wavelet is not shift invariant, it is better to apply the dual-tree complex wavelet in the ridgelet transform so that we can have what we call *complex ridgelets*. This can be done by replacing the 1D scalar wavelet with the 1D dual-tree complex wavelet transform in the last step of the ridgelet transform. In this way, we can combine the good property of the ridgelet transform with the approximate shift invariant property of the dual-tree complex wavelets.

The complex ridgelet transform can be applied to the entire image or we can partition the image into a number of overlapping squares and we apply the ridgelet transform to each square. We decompose the original $n \times n$ image into smoothly overlapping blocks of sidelength R pixels so that the overlap between two vertically adjacent blocks is a rectangular array of size $R/2 \times R$ and the overlap between two horizontally adjacent blocks is a rectangular array of size $R \times R/2$. For an $n \times n$ image, we count $2n/R$ such blocks in each direction. This partitioning introduces a redundancy of 4 times. In order to get the denoised complex ridgelet coefficient, we use the average of the four denoised complex ridgelet coefficients in the current pixel location.

The thresholding for the complex ridgelet transform is similar to the curvelet thresholding [10]. One difference is that we take the magnitude of the complex ridgelet coefficients when we do the thresholding. Let y_λ be the noisy ridgelet coefficients. We use the following hard thresholding rule for estimating the unknown ridgelet coefficients. When $|y_\lambda| > k\sigma\tilde{\sigma}$, we let $\hat{y}_\lambda = y_\lambda$. Otherwise, $\hat{y}_\lambda = 0$. Here, $\tilde{\sigma}$ is approximated by using Monte-Carlo simulations. The constant k used is dependent on the noise σ. When the noise σ is less than 30, we use $k = 5$ for the first decomposition scale and $k = 4$ for other decomposition scales. When the noise σ is greater than 30, we use $k = 6$ for the first decomposition scale and $k = 5$ for other decomposition scales.

The complex ridgelet image denoising algorithm can be described as follows:

1. Partition the image into $R \times R$ blocks with two vertically adjacent blocks overlapping $R/2 \times R$ pixels and two horizontally adjacent blocks overlapping $R \times R/2$ pixels

2. For each block, Apply the proposed complex ridgelets, threshold the complex ridgelet coefficients, and perform inverse complex ridgelet transform.

3. Take the average of the denoising image pixel values at the same location.

We call this algorithm $ComRidgeletShrink$, while the algorithm using the ordinary ridgelets $RidgeletShrink$. The computational complexity of $ComRidgeletShrink$ is similar to that of $RidgeletShrink$ by using the scalar wavelets. The only difference is that we replaced the 1D wavelet transform with the 1D dual-tree complex wavelet transform. The amount of computation for the 1D dual-tree complex wavelet is twice that of the 1D scalar wavelet transform. However, other steps of the algorithm keep the same amount of computation. Our experimental results show that $ComRidgeletShrink$ outperforms $VisuShrink$, $RidgeletShink$, and $wiener2$ filter for all testing cases. Under some case, we obtain 0.8dB improvement in Peak Signal to Noise Ratio (PSNR) over $RidgeletShrink$. The improvement over $VisuShink$ is even bigger for denoising all images. This indicates that $ComRidgeletShrink$ is an excellent choice for denoising natural noisy images.

3 Experimental Results

We perform our experiments on the well-known image *Lena*. We get this image from the free software package $WaveLab$ developed by Donoho et al. at Stanford University. Noisy images with different noise levels are generated by adding Gaussian white noise to the original noise-free images. For comparison, we implement *VisuShrink*, *RidgeletShrink*, *ComRidgeletShrink* and *wiener2*. *VisuShrink* is the universal soft-thresholding denoising technique. The $wiener2$ function is available in the MATLAB Image Processing Toolbox, and we use a 5×5 neighborhood of each pixel in the image for it. The $wiener2$ function applies a Wiener filter (a type of linear filter) to an image adaptively, tailoring itself to the local image variance. The experimental results in Peak Signal to Noise Ratio ($PSNR$) are shown in Table 1. We find that the partition block size of 32×32 or 64×64 is our best choice. Table 1 is for denoising image *Lena*, for different noise levels and a fixed partition block size of 32×32 pixels. The first column in these tables is the $PSNR$ of the original noisy images, while other columns are the PSNR of the denoised images by using different denoising methods. The $PSNR$ is defined as

$$PSNR = -10 \log_{10} \frac{\sum_{i,j}(B(i,j) - A(i,j))^2}{n^2 255^2}.$$

where B is the denoised image and A is the noise-free image. From Table 1 we can see that $ComRidgeletShrink$ outperforms $VisuShrink$, the ordinary *RidgeletShrink*, and $wiener2$ for all cases. *VisuShrink* does not have any denoising power when the noise level is low. Under such a condition, *VisuShrink* produces even worse results than the original noisy images. However, *ComRidgeletShrink* performs very well in this case. For some case, $ComRidgeletShrink$ gives us about 0.8 dB improvement over the ordinary $RidgeletShink$. This indicates that by combining the dual-tree complex wavelet into the ridgelet transform we obtain significant improvement in image denoising. The improvement of $ComRidgeletShrink$ over $VisuShrink$ is even more significant for all noisy levels and testing images. Figure 1 shows the noise free image, the image with noise added, the denoised image with *VisuShrink*, the denoised image with *RidgeletShrink*, the denoised image with *ComRidgeletShrink*, and the denoised image with *wiener2* for images *Lena*, at a partition block size of 32×32 pixels. It can be seen that $ComRidgeletShrink$ produces visually sharper denoised images than $VisuShrink$, the ordinary *RidgeletShrink*, and $wiener2$ filter, in terms of higher quality recovery of edges and linear and curvilinear features.

4 Conclusions and Future Work

In this paper, we study image denoising by using complex ridgelets. Our complex ridgelet transform is obtained by performing 1D dual-tree complex wavelet onto the Radon transform coefficients. The Radon transform is done by means of the projection-slice theorem. The approximate shift invariant property of the dual-tree complex wavelet transform makes the complex ridgelet transform an excellent choice for image denoising. The complex ridgelet transform provides near-ideal sparsity of representation for both smooth objects and objects with edges. This makes the thresholding of noisy ridgelet coefficients a near-optimal method of denoising for Gaussian white noise. We test our new denoising method with several standard images with Gaussian white noise added to the images. A very simple hard thresholding of the complex

Noisy Image	*VisuShrink*	*RidgeletShrink*	*ComRidgeletShrink*	*wiener2*
34.12	29.49	36.67	**37.19**	30.96
28.10	26.31	32.51	**33.13**	30.09
24.58	24.77	30.23	**30.79**	29.04
22.08	23.85	28.63	**29.21**	28.01
20.14	23.24	27.39	**28.00**	27.07
18.56	22.83	26.39	**27.03**	26.22

Table 1: The PSNR (dB) of the noisy images of Lena and the denoised images with different denoising methods.

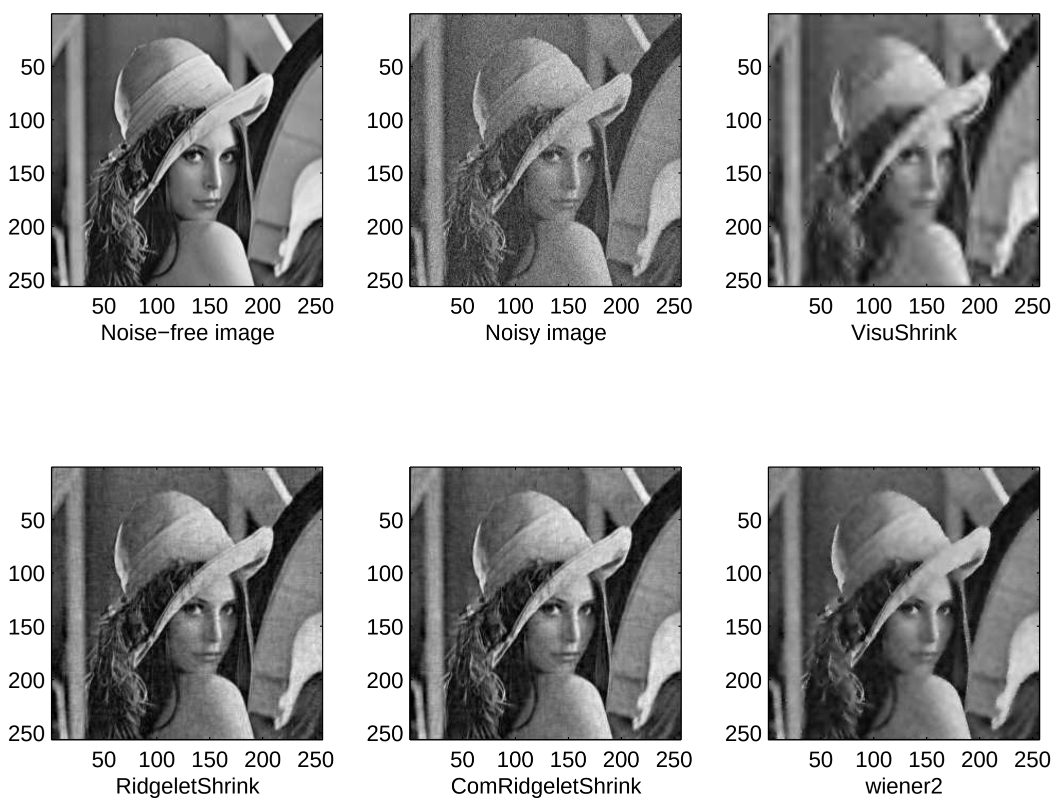

Figure 1: Image denoising by using different methods on a noisy image with PSNR = 22dB.

ridgelet coefficients is used. Experimental results show that complex ridgelets give better denoising results than *VisuShrink*, *wiener*2, and the ordinary ridgelets under all experiments. We suggest that *ComRidgeletShrink* be used for practical image denoising applications. Future work will be done by considering complex ridgelets in curvelet image denoising. Also, complex ridgelets could be applied to extract invariant features for pattern recognition.

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